Topic 1: Graphing Quadratic Equations (from Standard Form and Vertex Form)

Graph each equation using a table of values. Identify all key characteristics.

1. \( y = x^2 - 2x - 5 \)
   \[ x = \frac{-(-2)}{2(1)} = 1 \]

   \[
   \begin{array}{c|c}
   x & y \\
   \hline
   -1 & -2 \\
   0 & -5 \\
   1 & -6 \\
   2 & -5 \\
   3 & -2 \\
   \end{array}
   \]

   Axis of Symmetry: \( x = 1 \)
   Vertex: \((1, -6)\)

   Domain: \( \mathbb{R} \)
   Range: \( y \geq -6 \)

2. \( y = -x^2 + 10x - 28 \)
   \[ x = \frac{-10}{2(-1)} = 5 \]

   \[
   \begin{array}{c|c}
   x & y \\
   \hline
   3 & -7 \\
   4 & -4 \\
   5 & -3 \\
   6 & -4 \\
   7 & -7 \\
   \end{array}
   \]

   Axis of Symmetry: \( x = 5 \)
   Vertex: \((5, -3)\)

   Domain: \( \mathbb{R} \)
   Range: \( y \leq -3 \)

3. \( y = 2x^2 + 4x \)

   \[
   \begin{array}{c|c}
   x & y \\
   \hline
   -3 & 6 \\
   -2 & 0 \\
   -1 & -2 \\
   0 & 0 \\
   1 & 6 \\
   \end{array}
   \]

   Axis of Symmetry: \( x = -1 \)
   Vertex: \((-1, -2)\)

   Domain: \( \mathbb{R} \)
   Range: \( y \geq -2 \)

4. \( y = -x^2 + 7 \)

   \[
   \begin{array}{c|c}
   x & y \\
   \hline
   -2 & 3 \\
   -1 & 1 \\
   0 & 1 \\
   1 & 3 \\
   2 & 1 \\
   \end{array}
   \]

   Axis of Symmetry: \( x = 0 \)
   Vertex: \((0, 7)\)

   Domain: \( \mathbb{R} \)
   Range: \( y \leq 7 \)

5. \( y = (x + 3)^2 - 8 \)

   \[
   \begin{array}{c|c}
   x & y \\
   \hline
   -5 & -4 \\
   -4 & -7 \\
   -3 & -8 \\
   -2 & -7 \\
   -1 & -4 \\
   \end{array}
   \]

   Axis of Symmetry: \( x = -3 \)
   Vertex: \((-3, -8)\)

   Domain: \( \mathbb{R} \)
   Range: \( y \geq -8 \)

6. \( y = -3(x - 1)^2 \)

   \[
   \begin{array}{c|c}
   x & y \\
   \hline
   -1 & -12 \\
   0 & -3 \\
   1 & 0 \\
   2 & -3 \\
   3 & -12 \\
   \end{array}
   \]

   Axis of Symmetry: \( x = 1 \)
   Vertex: \((1, 0)\)

   Domain: \( \mathbb{R} \)
   Range: \( y \leq 0 \)
### Topic 2: Vertex Form & Transformations

**Describe the transformations from the parent function given each equation.**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Transformation</th>
</tr>
</thead>
<tbody>
<tr>
<td>7. $y = -x^2 + 6$</td>
<td>Reflected across the X-axis, up 6</td>
</tr>
<tr>
<td>8. $y = (x + 4)^2 - 1$</td>
<td>Left 4 and Down 1</td>
</tr>
<tr>
<td>9. $y = 2(x - 5)^2 + 4$</td>
<td>Vertical stretch by 2, Right 5, up 4</td>
</tr>
</tbody>
</table>

10. If the graph of the function $y = x^2$ is reflected over the x-axis, then translated two units left, write an equation to represent the function.

$$y = -(x + 2)^2$$

11. If the graph of the function $y = x^2$ is vertically compressed by a factor of $\frac{1}{4}$, then translated seven units right and one unit down, write an equation to represent the function.

$$y = \frac{1}{4}(x - 7)^2 - 1$$

### Topic 3: Quadratic Roots (Zeros)

**Graph each function, identify the zeros, then write the equation in factored form, if possible.**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Graph</th>
<th>Zeros</th>
<th>Factored Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>12. $y = x^2 + 8x + 15$</td>
<td><img src="image1" alt="Graph" /></td>
<td>$x = -5, -3$</td>
<td>$y = (x+5)(x+3)$</td>
</tr>
<tr>
<td>13. $y = -2x^2 + 8x - 8$</td>
<td><img src="image2" alt="Graph" /></td>
<td>$x = \frac{3}{2}$</td>
<td>$y = -2(x-2)(x+2)$</td>
</tr>
<tr>
<td>14. $y = -x^2 - 1$</td>
<td><img src="image3" alt="Graph" /></td>
<td>-</td>
<td>Not possible</td>
</tr>
</tbody>
</table>

15. $y = (x+1)^2 - 4$

- $y = (x+1)(x+1) - 4$
- $y = x^2 + 2x - 3$

Factored Form:

$y = (x+3)(x-1)$

Zeros: $x = -3, 1$

16. $y = 2(x-3)^2 - 18$

- $y = 2(x-3)(x-3) - 18$
- $y = 2(x^2 - 6x + 9) - 18$
- $y = 2x^2 - 12x$

Factored Form:

$y = 2(x-3)(x+3)$

Zeros: $x = 3, 6$

17. $y = -(x+5)^2 + 9$

- $y = -(x+5)(x+5) + 9$
- $y = -(x^2 + 10x + 25) + 9$
- $y = -x^2 - 10x - 16$

Factored Form:

$y = -(x+5)(x-3)$

Zeros: $x = -5, 3$
### Find the discriminant of each equation. Then, determine the number of solutions.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Discriminant</th>
<th>Number of Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>18. $y = -x^2 + 7x - 15$</td>
<td>$7^2 - 4(-1)(-15)$</td>
<td>2</td>
</tr>
<tr>
<td>19. $y = 3x^2 - 12x$</td>
<td>$(12)^2 - 4(3)(0)$</td>
<td>1</td>
</tr>
<tr>
<td>20. $y = x^2 - 20x + 100$</td>
<td>$(-20)^2 - 4(1)(100)$</td>
<td>0</td>
</tr>
</tbody>
</table>

### Topic 4: Solving Quadratic Equations

#### Solve each equation. Simplify all irrational solutions.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>21. $x^2 + 4x - 45 = 0$</td>
<td>$x = -9, 5$</td>
</tr>
<tr>
<td>22. $2x^2 - 9 = 39$</td>
<td>$x = \pm 15$</td>
</tr>
<tr>
<td>23. $x^2 - 10x - 3 = 0$</td>
<td>$x = \frac{5 \pm 2\sqrt{17}}{2}$</td>
</tr>
<tr>
<td>24. $16x^2 = 10x$</td>
<td>$x = \frac{5}{8}$</td>
</tr>
<tr>
<td>25. $3x^2 - 8x - 8 = 0$</td>
<td>$x = \frac{8 \pm \sqrt{(8)^2 - 4(3)(-8)}}{2(3)} = \frac{8 \pm \sqrt{144}}{6} = \frac{8 \pm 12}{6} = \frac{4 \pm 2\sqrt{10}}{3}$</td>
</tr>
<tr>
<td>26. $-x^2 + 3x = x - 19$</td>
<td>$x = \frac{-2 \pm \sqrt{2^2 - 4(-1)(19)}}{2(-1)} = \frac{-2 \pm \sqrt{4 + 76}}{-2} = \frac{-2 \pm 2\sqrt{19}}{-2} = \frac{1 \pm \sqrt{19}}{1}$</td>
</tr>
</tbody>
</table>
27. \( x^2 - 2x - 17 = 0 \)
\begin{align*}
&X^2 - 2x = 17 \\
&(-1)^2 = 1 \\
&X^2 - 2x + 1 = 17 + 1 \\
&\sqrt{(x-1)^2} = \sqrt{18} \\
&x - 1 = \pm \sqrt{18} \\
&\boxed{x = \left\{ \pm 3\sqrt{2} \right\}}
\end{align*}

28. \( 6x^2 = 7x + 5 \)
\begin{align*}
&6x^2 - 7x - 5 = 0 \\
&x^2 - 7x - 30 = 0 \\
&(x - 10)(x + 3) = 0 \\
&\frac{10}{6} \\
&(3x - 5)(2x + 1) = 0 \\
&3x = \frac{5}{3} \\
&2x + 1 = 0 \\
&x = -\frac{1}{2} \\
&\boxed{x = \left\{ -\frac{5}{3}, \frac{5}{3} \right\}}
\end{align*}

29. \( 2x^2 + 19 = 1 - 20x \)
\begin{align*}
&2x^2 + 20x + 18 = 0 \\
&2(x^2 + 10x + 9) = 0 \\
&2(x + 1)(x + 9) = 0 \\
&2 \neq 0 \\
&x + 1 = 0 \\
&x = -1 \\
&x = -9
\end{align*}
\[ \boxed{x = \left\{ -9, -1 \right\}} \]

30. \( 25x^2 + 1 = 5 \)
\begin{align*}
&25x^2 = 4 \\
&\frac{25}{25} \\
&\sqrt{x^2} = \sqrt{1} \\
&x = \pm \frac{2}{5} \\
&\boxed{x = \left\{ -\frac{2}{5}, \frac{2}{5} \right\}}
\end{align*}

31. \( \frac{1}{2} x^2 - 42 = 8 \)
\begin{align*}
&2 \cdot \frac{1}{2} x^2 = 50 \cdot 2 \\
&\sqrt{x^2} = \sqrt{100} \\
&x = \pm 10
\end{align*}
\[ \boxed{x = \left\{ -10, 10 \right\}} \]

32. \( x^2 + 9x + 13 = 4 \)
\begin{align*}
&x^2 + 9x + 9 = 0 \\
&x = -9 \pm \sqrt{9^2 - 4(1)(9)} \\
&2(1) \\
&x = -9 \pm \sqrt{81 - 36} \\
&2 \\
&x = -9 \pm \frac{45}{2} \\
&\boxed{x = \left\{ -9 \pm 3\sqrt{5} \right\}}
\end{align*}

---

**Topic 5: Area and Consecutive Integer Problems**

33. If the area of the rectangle below is 42 inches squared, find the value of \( x \).

\[ \boxed{x = 6 \text{ in}} \]

\[ x + 8 \]
\[ (x + 8)(x - 3) = 42 \]
\[ x^2 + 5x - 24 = 42 \]
\[ x^2 + 5x - 66 = 0 \]
\[ (x + 11)(x - 6) = 0 \]
\[ x = -11, x = 6 \]
\[ x = 6 \]

---

34. The length of a rectangle is five feet less than its width. If the area of the rectangle is 84 square feet, find its dimensions.

\[ \text{let } x = \text{width} \]
\[ x - 5 = \text{length} \]
\[ x(x - 5) = 84 \]
\[ x^2 - 5x - 84 = 0 \]
\[ (x - 12)(x + 7) = 0 \]
\[ x = 12 \]
\[ \boxed{12 \text{ ft, } 7 \text{ ft}} \]

\[ x = -7 \]
35. A square was altered so that one side is increased by 9 inches and the other side is decreased by 2 inches. The area of the resulting rectangle is 60 square inches. What was the area of the original square?

Let \( x = \) square side
\[ x + 9 \] (rectangle sides)
\[ x - 2 \]

\[
(x + 9)(x - 2) = 60 \\
x^2 + 7x - 18 = 0 \\
x = \frac{7 \pm \sqrt{49 + 4 \cdot 18}}{2} \\
x = \frac{7 \pm 13}{2}
\]

\( x = 6 \) \( x = 9 \)

\[ \text{Area} = 6^2 = 36 \text{ in}^2 \]

36. Find two consecutive positive integers such that the sum of their squares is 145.

Let \( x = 1^{st} \) consecutive integer
\( x + 1 = 2^{nd} \) consecutive integer

\[
x^2 + (x+1)^2 = 145 \\
x^2 + x^2 + 2x + 1 = 145 \\
2x^2 + 2x + 1 = 145 \\
2x^2 + 2x - 144 = 0 \\
2(x^2 + x - 72) = 0 \\
2(x + 9)(x - 8) = 0 \\
2 = 0 \quad x = 9 \quad x = 8
\]

8, 9

37. Natalie found a tennis ball outside a tennis court. She picked up the ball and threw it over the fence into the court. The path of the ball can be represented by the equation \( h = -16t^2 + 18t + 5 \).

a. Find the maximum height of the tennis ball.

\[
t = \frac{-(-18)}{2(-16)} = 0.56 \text{ s}
\]

\[
h = -16(0.56)^2 + 18(0.56) + 5 = 10.06 \text{ ft}
\]

b. How long will it take to reach the ground?

\[
t = \frac{-(-18) \pm \sqrt{(-18)^2 - 4(-16)(5)}}{2(-16)}
\]

\[
= \frac{-(-18) \pm \sqrt{324 + 320}}{-32}
\]

\[
= \frac{-(-18) \pm 1.36 \text{ sec}}{-32}
\]

1.36 sec

38. A circus acrobat is shot out of a cannon with an initial upward speed of 50 ft/s. The equation for the acrobat’s pathway can be modeled by the equation \( h = -16t^2 + 50t + 4 \).

a. Find the maximum height of the acrobat.

\[
t = \frac{-50}{2(-16)} = 1.56 \text{ s}
\]

\[
h = -16(1.56)^2 + 50(1.56) + 4 = 43.06 \text{ ft}
\]

b. How long will it take to reach the ground?

\[
t = \frac{-50 \pm \sqrt{50^2 - 4(-16)(4)}}{2(-16)}
\]

\[
= \frac{-50 \pm \sqrt{2500 - 32}}{-32}
\]

\[
= \frac{-50 \pm 15.6}{-32}
\]

\[
= -0.86, 3.20 \text{ s}
\]

3.20 sec

39. Kate recorded the time it took six children of different ages to run one lap around the track. Write an equation for the line of best fit, then estimate the length of time it would take for her 5-year old brother to run one lap.

<table>
<thead>
<tr>
<th>Age (yrs)</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>180</td>
</tr>
<tr>
<td>11</td>
<td>137</td>
</tr>
<tr>
<td>12</td>
<td>126</td>
</tr>
<tr>
<td>12</td>
<td>124</td>
</tr>
<tr>
<td>15</td>
<td>102</td>
</tr>
<tr>
<td>16</td>
<td>78</td>
</tr>
</tbody>
</table>

\[
y = -12.91x + 285.88
\]

\[
y = -12.91(5) + 285.88 = 221.33 \text{ sec}
\]

\[221 \text{ sec}\]

40. A pistol is accidentally discharged vertically in the air. The height, \( h \), of the bullet at time \( t \) seconds is recorded in the table below. Using an equation for the curve of best fit, find the height of the pistol after 10 seconds.

<table>
<thead>
<tr>
<th>( t ) (s)</th>
<th>( h ) (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>187</td>
</tr>
<tr>
<td>2</td>
<td>339</td>
</tr>
<tr>
<td>3</td>
<td>459</td>
</tr>
<tr>
<td>4</td>
<td>547</td>
</tr>
</tbody>
</table>

\[
y = -16x^2 + 200x + 3
\]

\[
y = -16(10)^2 + 200(10) + 3 = 403 \text{ ft}
\]

\[403 \text{ ft} \]